Roll No. ....

Subject Code—4390

M. Sc. EXAMINATION

(Second Semester)

(Main)

MATHEMATICS

MAL-525

Complex Analysis–II

Time : 3 *Hours* *Maximum Marks* : 100

Note : Attempt any *Five* questions. All questions carry equal marks.

1. (a) State and prove Reimann mapping theorem.

(b) (i) State and prove Hurwitz's theorem.

(ii) Define Mean Value Property for a continuous function. Also define subharmonic and superharmonic function.

2. State and prove Weierstrass factorization theorem.

3. (a) Define Zeta function of Reimann and prove that :

$$\varsigma(s)\,\Gamma(s) = \int_0^\infty \frac{t^{s-1}}{e^t - 1}\,dt \text{ for Re } s > 1.$$

(b) Define Gamma function and prove the following :

(i) $\Gamma\left(\frac{1}{2}+z\right)\Gamma\left(\frac{1}{2}-z\right) = \frac{\pi}{\cos \pi z}$.

(ii) $\Gamma(z) = \lim\limits_{n\to\infty} \frac{n!n^z}{(z+1)(z+2).....(z+n)}$

4. (a) Using Hadamard's factorization theorem show that :

$$\sin \pi z = \pi z \prod_{n=1}^{\infty}\left(1-\frac{z^2}{n^2}\right).$$

(b) State and prove Montel-Caratheodory theorem.

5. (a) Define order and type of an entire function. Show that the order ρ of an entire function is given by

$$\rho = \overline{\lim_{r\to\infty}} \frac{\log \log M(r)}{\log r}.$$

(b) State and prove Jensen's formula.

6. State and prove Schottky's theorem.

7. (a) State and prove Great Picard Theorem.

(b) Let G be a region with the metric space Har(G) is complete and $\{u_n\}$ is a sequence in Har(G) such that $u_1 \le u_2 \le ......$, then either $u_n(z) \to \infty$ univformly on compact subsets of G or $\{u_n\}$ converges in Har(G) to a harmonic function. (Harnack's Theorem).