Roll No. ....

Subject Code—4389

M. Sc. EXAMINATION

(Second Semester)

(Main)

MATHEMATICS

MAL-524

Ordinary Differential Equations–II

Time : 3 *Hours* *Maximum Marks* : 100

Note : Attempt any *Five* questions. All questions carry equal marks.

1. (a) Define a linear system. Prove that the set of cell solutions of a linear homogeneous system on an interval form a finite dimensional vector space over complex field. What is the dimension of this vector space ? Justify your answer. 10

(b) If Φ is a fundamental matrix for a periodic system : 10

$$x'(t) = A(t)x(t),$$

then $\Phi(t + w)$ is also a fundamental matrix. For each such Φ, show that there exists a periodic non-singular matrix P of period w and a constant matrix R s.t. : 10

$$\Phi(t) = P(t)\, e^{tR}$$

2. (a) Describe method of reduction of order for solving a linear homogeneous system. 10

(b) Determine fundamental Matrix e^{tA} for

$$x'(t) = Ax(t), \text{ where } A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & -2 & 3 \\ 0 & 1 & 0 \end{pmatrix}.$$ 10

3. (a) Prove that a necessary and sufficient condition for the system : 10

$$x'(t) = Ax(t)$$

to have a non-zero periodic solution of period w is that $I - e^{Aw}$ is singular, where I is the identity matrix.

(b) Determine the type and stability of critical point of the system : 10

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} -3 & -4 \\ 4 & -9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

4. (a) If the roots of the characteristics equation of the system : 12

$$\frac{dx}{dt} = ax + by$$

$$\frac{dy}{dt} = cx + dy$$

are pure imaginary, then prove that origin is centre.

(b) What are the critical points of the system : 10

$$\dot{x} = x + 4y - x^2$$

$$\dot{y} = 6x - y + 2xy$$

Find their type and discuss stability.

5. (a) Construct Liapunov function for the system : 10

$$\frac{dx}{dt} = -x + 2x^2 + y^2$$

$$\frac{dy}{dt} = -y + xy$$

(b) State and prove Benedixson Non-existence Theorem. 10

6. (a) Find the curve joining given two points which is traversed by a particle moving under gravity from one point to another in shortest time. The resistance of medium can be neglected. 12

(b) Derive Euler's equations when a functional depends on functions of several independent variables. 8

7. (a) Find the shortest distance between the parabola $y = x^2$ and the line $x - y = 5$. 10

(b) Explain variational problems in parametric form. 10

8. (a) Explain method for finding extremum of

$$\int_a^b F\left(x, y, y', y'', \ldots\ldots, y^{(n)}(x)\right) dx.$$ 12

(b) Find extremals of the functional : 8

$$\int_a^b \left\{2xy + \left(y'''\right)^2\right\} dx$$