Roll No. ....

Subject Code—4388

M. Sc. EXAMINATION

(Second Semester)

(Main)

MATHEMATICS

MAL-523

Methods of Applied Mathematics

Time : 3 Hours *Maximum Marks : 100*

Note : Attempt any *Five* questions. All questions carry equal marks.

1. (a) Define Fourier transform and evaluate it for the function :

$$f(t) = \begin{cases} 1, & |t| \leq a \\ 0, & \text{otherwise} \end{cases}$$

Hence evaluate

$$\int_{-\infty}^{\infty} \frac{\sin (sa) \cos (st)}{s} \, ds .$$

(b) Find Fourier cosine transform of :

$$f(t)=\frac{1}{1+t^2}.$$

2. (a) Using Fourier transforms, solve :

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, x>0, t>0.$$

subject to conditions :

(i) $u = 0$, when $x = 0$, $t > 0$

(ii) $u=\begin{cases}1, & 0<x<1\\ 0, & x\geq 1\end{cases}$ when $t = 0$

and $u(x,t)$ is bounded.

(b) Obtain expression for curl $\vec{F}$ in orthogonal curvilinear co-ordinates and deduce it in cylindrical coordinates.

3. (a) Represent the vector :

$$\vec{A}=z\hat{i}-2x\hat{j}+y\hat{k}$$

in spherical co-ordinates.

(b) Let $\vec{A}$ be a given vector defined in two general curvilinear co-ordinate systems (u_1,u_2,u_3) and $(\overline{u}_1,\overline{u}_2,\overline{u}_3)$. Find the relation between the covariant components of the vector in the two co-ordinate systems.

4. (a) Define a random variable and mathematical expectation. A continuous random variable X has density function $f(x)=3x^2$, $0\leq x\leq 1$. Find 'a' and 'b' such that $P(X \leq a) = P(X > a)$ and $P(X > b) = 0{\cdot}05$.

(b) Define moment generating function. If X assumes the value 'r' with probability $P(X = r) = pq^{r-1}$, $r = 1, 2, ...$ Find moment genrating function and hence mean and variance.

5. (a) Prove the following for Binomial distribution :

$$\mu_{r+1} = pq\left[n.r\mu_{r-1} + \frac{d\mu_r}{dp}\right].$$

(b) Prove that all cumulants are equal for a Poisson distribution. Also prove that moment generating function of Binomial distribution tends to the m.g.f. of the Poisson distribution as $n \to \infty$ and $np = m$ is finite.

6. (a) Define geometric distribution. Derive expression its M.G.E. and hence find its mean and variance.

(b) Prove that in a Normal distribution, the Mean, Media and Mode coincide.

7. (a) Prove that for a Normal distribution, the standard deviation is the distance from the axis of symmetry to a point of Inflexion.

(b) Prove that :

$$\sigma_{1\cdot23}^2 = \frac{\omega}{\omega_{11}}\sigma_1^2$$

with usual notations.

8. (a) Define Partial correlation and prove that :

$$r_{12\cdot3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{\left(1-r_{13}^2\right)\left(1-r_{23}^2\right)}}$$

(b) Define t-distribution and obtain its Mean and Variance.