Roll No. ....

Subject Code—4387

M. Sc. EXAMINATION

(Second Semester)

(Main)

MATHEMATICS

MAL-522

MEASURE AND INTEGRATION THEORY

Time : 3 Hours *Maximum Marks :* 100

Note : Attempt any *Five* questions. All questions carry equal marks.

1. (a) Let f and g be measurable functions and c be a constant. Then show that $f + c$, cf, $f + g$, $|f|$, f^2, fg are all measurable. **10**

(b) Let f and g be functions such that $f = g$ a.e. If f is measurable, then g is also measurable. **5**

(c) Show that a characteristic function χ_A is measurable iff A is measurable. **5**

2. (a) Prove that every measurable function can be approximated by a sequence of simple functions. 15

(b) State and prove F. Riesz theorem for convergence in measure. 5

3. (a) State and prove Lusin's theorem. 12

(b) Prove that almost uniform convergence implies convergence in measure. Also prove by an example that convergence in measure does not necessarily imply convergence pointwise at any point. 8

4. (a) Let f be bounded on a measurable set E with $m(E) < \infty$. Show that f is Lebesgue integrable iff f is measurable. 10

(b) Let f be a bounded function defined on $[a, b]$. Then if f is Riemann integrable, then it is Lebesgue integrable. Discuss the convex also. 10

5. If f and g are bounded measurable functions defined on a set E of finite measure, then show that : 20

(i) $\int_E af = a\int_E f$

(ii) $\int_E (f+g) = \int_E f + \int_E g$

(iii) If $f \le g$ a.e., then $\int_E f \le \int_E g$

(iv) If $f = g$ a.e., then $\int_E f = \int_E g$

(v) If $A \le f(r) \le B$, then

$$A\,m(E) \le . \int_E f \le B\,m(E)$$

(vi) If A and B are disjoint measurable sets of finite measure, then :

$$\int_{A\cup B} f = \int_A f + \int_B f$$

6. (a) State Fatou's lemma. Show that strict inequality may occur in Fatou's lemma. 6

(b) Let f be a measurable function over E. Then f is integrable over E iff $|f|$ is integrable over E. Also $\left|\int_E f\right| \le \int_E |f|$. 6

(c) State and prove Lebesgue bounded convergence theorem. 8

7. (a) Show that every non-decreasing function f defined on the interval $[a, b]$ is differentiable almost everywhere in $[a, b]$. Also f' is measurable and : 10

$$\int_a^b f'(x)dx \leq f(b)-f(a)$$

(b) State and prove Vitali covering lemma. 10

8. (a) Let f be an integrable function on $[a, b]$ and let $F(x) = F(a) + \int_a^x f(t)dt$. Then $F'(x) = f(x)$ for almost all x in $[a, b]$. 10

(b) If f is absolutely continuous on $[a, b]$ and $f'(x) = 0$, a.e. then f is constant. 10