

Roll No.

Subject Code—4386

M. Sc. EXAMINATION

(Second Semester)

(Main)

MATHEMATICS

MAL-521

Abstract Algebra

Time : 3 Hours

Maximum Marks : 100

Note : Attempt any *Five* questions. All questions carry equal marks.

1. If V is finite dimensional vector space over F and all the roots of minimal polynomial of $T \in A(V)$ lies in F , then T satisfies a polynomial of degree n over F . 20

2. (a) If $T \in A(V)$ is nilpotent, then show that :

$$1 + \alpha_1 T + \alpha_2 T^2 + \dots + \alpha_r T^r$$

is invertible. 8

- (b) If $\lambda \in F$ is a characteristic root of T , then for any polynomial $q(x)$ over $F[x]$, $q(\lambda)$ is a characteristic root of $q[T]$. 12

3. (a) If subspace W of vector space is invariant under T , then T induces a linear

transformation \bar{T} on $\frac{V}{W}$, define by

$(v + W)\bar{T} = vT + W$. Also show that minimal polynomial of \bar{T} divides the minimal polynomial of T . 13

- (b) Let $\lambda \in F$ is a characteristic root of $T \in A(V)$. If $ST = TS$, $S \in A(V)$ then show that set of all characteristic vectors, associated with λ , is invariant under T . 7

4. (a) Show that invariant of a nilpotent transformations are unique. 10

- (b) Let $p(x)$ be the minimal polynomial of $T \in A(V)$, V is cyclic module (cyclic w.r.t. T). Then show that there always exist a basis of V in which the matrix of T is companion matrix of $p(x)$. 10

5. Define Artinian and Noetherian rings. Give an example of each with explanation : 20

- (a) Noetherian but not Artinian
(b) Artinian but not Noetherian
(c) Neither Noetherian nor Artinian.

6. (a) Let M be an left R -module and N be a submodule of M . Then M is artinian iff

both N and $\frac{M}{N}$ are Artinian. 14

- (b) Does the Hilbert basis theorem holds for artinian rings also. 6

7. (a) Let M be finitely generated free module over a commutative ring R . Then all the basis of M has same number of element. 13

- (b) Prove that quotient module of an semi-simple module is semi-simple. 7

8. Define finitely co-generated module. Prove that for a left R -module M , following conditions are equivalent : 20

- (a) M is Artinian
- (b) Every non-empty family of R -module has a minimal element
- (c) Every quotient module of M is finitely co-generated.