Roll No. ....

Subject Code—3229

M. Sc. EXAMINATION

(First Semester)

MATHEMATICS

MAL-515

Complex Analysis–I

Time : 3 Hours *Maximum Marks : 100*

Note : Attempt any *Five* questions. All questions carry equal marks.

1. (a) Derive the polar form of the Cauchy-Riemann equations.

(b) For what values of z the function defined by the following equations ceases to be analytic :

$$z = \sinh u \cos v + i \cosh u \sin v$$

where $w = u + iv$.

2. State and prove Cauchy-Goursat theorem.

3. (a) Define entire function. State and prove Liouville's theorem.

(b) If $f(z)$ is analytic within and on a simple closed contour C, then show that the point giving the maximum of $|f(z)|$ can lie on the boundary C and not within it.

4. (a) State and prove Schwarz's lemma.

(b) Define branch of a logarithm function $\log z : z \in C$. Show that it is analytic and its derivative is $\frac{1}{z}$.

5. (a) Define a conformal mapping. Prove that at each point z of a domain, where $f(z)$ is analytic and $f'(z) \neq 0$ the mapping $w = f(z)$ is conformal.

(b) Show that the set of all bilinear transformation forms a non-abelian group under the product of transformations.

6. (a) Show that :

$$\exp\left[\frac{c}{2}\left(z-\frac{1}{z}\right)\right] = \sum_{n=-\infty}^{\infty} a_n z^n$$

where $a_n = \frac{1}{2\pi}\int_0^{2\pi} \cos(n\theta - c\sin\theta)\, d\theta$.

(b) Define isolated, non-isolated, removable singularities. Also show that the poles of a function are isolated.

7. (a) State and prove Argument principle.

(b) State Rouche's theorem and apply it to prove that if $a > e$, the equation $e^z = az^n$ has n roots inside the circle $|z| = 1$.

8. (a) Define residue at poles and residue at infinity. State Cauchy's residue theorem.

Also use it to evaluate $\int_C \frac{e^z dz}{z(z-1)^2}$, where C is the circle $|z| = 2$.

(b) Evaluate by residue theorem $\int_0^{\pi} \frac{a\,d\theta}{a^2 + \sin^2\theta}$, where $a > 0$.