Roll No. ....

Subject Code—3228

M. Sc. EXAMINATION

(First Semester)

MATHEMATICS

MAL-514

Ordinary Differential Equations–I

Time : 3 Hours *Maximum Marks :* 100

Note : Attempt any *Five* questions. All questions carry equal marks. Scientific calculator may be allowed to students.

1. (a) Define Initial-Value Problem (IVP). State and prove the relation between the solution of IVP and the corresponding integral equation. 10

(b) State and prove Cauchy-Euler construction theorem for an approximation solution of IVP. 10

2. (a) State and prove Picard-Lindelof theorem. 14

(b) Find the first four Picard successive approximations of the initial value problem : 6

$$x'(t) = x + t, \quad x(0) = 1$$

Find the nth approximation and its limit.

3. (a) State and prove Extension Theorem. 12

(b) Given that :

$$\frac{dy}{dx} = \log(x+y), \; y(0) = 2$$

Using Modified Euler's method, find an approximate value of y when $x = 1{\cdot}2$ and $1{\cdot}4$ with $h = 0{\cdot}2$. 8

4. (a) State and prove the basic theorem concerning the dependence of solution of IVP on function. 12

(b) Reduce the general n-th order initial value problem to an equivalent vector matrix differential equation. 8

5. (a) Prove that, a necessary and sufficient condition that there exists between two functions $u(x, y)$ and $v(x, y)$ a relation $F(u, v) = 0$, not involving x or y explicitly is that : 10

$$\frac{\partial(u, v)}{\partial(x, y)} = 0.$$

(b) Verify that the equation : 10

$$z(z+y^2)dx + z(z+x^2)dy - xy(x+y)dz = 0$$

is integrable and find its primitive using Natani's method.

6. (a) State and prove the comparison theorem for differential inequations. 10

(b) State and prove Sturm's Fundamental Comparison theorem. 10

7. (a) Explain Riccati equation. Hence, solve the IVP : 10

$$\frac{dy}{dx} = \frac{2\cos^2 x - \sin^2 x + y^2}{2\cos x}, \; y(0) = -1$$

(b) Explain the phase plane method to seek the solution of Sturm-Liouville equation. **10**

8. (a) Prove that eigenvalues of a Sturm-Liouville Boundary Value Problem are real and discrete. **10**

(b) Find the eigenvalues and eigenfunctions of the Sturm-Liouville problem : **10**

$$\frac{d^2y}{dx^2} = +\lambda y = 0, \; y(0) = 0, \; y'(\pi) = 0.$$