Roll No. ....

Subject Code—3225

M. Sc. EXAMINATION

(First Semester)

MATHEMATICS

MAL-511

Algebra

Time : 3 Hours *Maximum Marks :* 100

Note : Attempt any *Five* questions. All questions carry equal marks.

1. (a) State and prove Zassenhaus Lemma.

(b) Define normal and subnormal series. Prove that every normal series is subnormal but converse may not be true.

2. (a) Define nilpotent group. Prove that every p-group is nilpotent.

(b) Prove that any two subnormal series have equivalent refinements.

3. Define solvable groups. Prove that subgroups and factor groups of a solvable groups are solvable.

4. (a) Prove that if L is finite extension of K and K is a subfield of L which contains F, then [K : F] | [L : F].

   (b) If a and b are algebraic over F of degree m and n respectively and if m and n are relatively prime, prove that F(a, b) is of degree mn over F.

5. (a) Prove that regular septagon is not constructible by straight edge and compass.

   (b) Prove that a polynomial of degree $n \geq 5$ is not solvable by radicals.

6. (a) Prove that K is normal extension of F iff K is the splitting field of some polynomial over F.

   (b) Find Galois group of polynomial $x^4 - x^2 - 6$ over the field of rational numbers.

7. (a) Define Perfect fields, Separable fields and Finite fields with examples. Show that for given prime p and positive integer m, there always exist a finite field of order p^m.

   (b) Define algebraically closed field with example.

8. Define Prime Field. Prove that Q (field of rational numbers) and Z_p, field of integers modulo p are the prime fields. Also prove that every field is either isomorphic to Q or to Z_p.