Roll No. .....

Subject Code—3227

M. Sc. EXAMINATION

(First Semester)

MATHEMATICS

MAC-513

Mechanics

Time : 3 *Hours* *Maximum Marks :* 100

Note : Attempt any *Five* questions. All questions carry equal marks.

1. (a) State and prove parallel axes theorem for moment of inertia.

   (b) Define equimomental systems. Derive the necessary and sufficient conditions for two systems to be equimomental.

2. (a) Define principal axis for a point of a rigid body. Find the directions of the principal axes at one corner of a uniform rectangular lamina of dimensions $2a \times 2b$ and density ρ.

(b) What do you mean by Holonomic and non-holonomic dynamical systems ? Derive Lagrange's equations of motion for a particle moving under the inverse square law of attraction $\frac{\mu m}{r^2}$.

3. (a) Explain the notions of generalized velocities and generalised momentum. Show that the generalised momentum increment is equal to the generalised impulsive force associated with each generalised co-ordinate.

(b) Show that kinetic energy of a holonomic system is a quadratic function of the generalised velocities. Deduce the result $p_1\dot{q}_1 + p_2\dot{q}_2 + ... + p_n\dot{q}_n = 2T$, when time is explicitly absent and T is kinetic energy.

4. (a) Explain the following :

(i) Generalised potential and cyclic co-ordinates

(ii) Hamilton's canonical equations

(iii) Poisson's bracket.

(b) State and derive Poisson's identity for Poisson's brackets.

5. (a) State and prove Jacobi-Poisson Theorem.

(b) State and prove principle of least action. Also state major difference between this and Hamilton's principle.

6. (a) State and derive Whittaker's equations.

(b) Define canonical transformations. Prove that Poisson's bracket is invariant under canonical transformations.

7. (a) Derive Hamilton-Jacobi equation. Use it to solve the problem of one-dimensional linear harmonic oscillator.

(b) Find the potential at any point inside the thin spherical shell. Show that this is constant and is equal to potential on the surface of the shell.

8. (a) Define attraction of a field. Find attraction and potential at a point external to uniform rod.

(b) Write a short note on Surface and Solid Harmonics.