2007

Subject Code—7271

M.C.A. (First Year) EXAMINATION

MATHEMATICS—I

MCA-103

Time : 3 *Hours* *Maximum Marks* : 100

Note : Attempt any *Five* questions out of the eight set. All questions carry equal marks.

1. (a) By finding A^{-1} solve the linear equation

$$AX = B, \text{ where } A = \begin{bmatrix} 3 & 4 & 5 \\ 1 & 2 & 0 \\ 5 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

and $B = \begin{bmatrix} 4 \\ -1 \\ 5 \end{bmatrix}$.

(b) Prove that :

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3.$$

2. (a) Prove that :

(i) $\sin 10^\circ \sin 50^\circ \sin 60^\circ \sin 70^\circ = \sqrt{3}/16$

(ii) $\tan 50^\circ = \tan 40^\circ + 2\tan 10^\circ$

(b) If $\cos\theta = \dfrac{\cos\phi - e}{1 - e\cos\phi}$, show that :

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1+e}{1-e}}\tan\frac{\phi}{2}.$$

3. (a) Find the distance between the lines $9x + 40y - 20 = 0$ and $9x + 40y + 103 = 0$.

(b) Find the equations of the two lines which can be drawn through the point (2, 2) to make an angle of 45° with the line $x + y = 2$.

(c) Find the condition that the points $P\left(at_1, \dfrac{a}{t_1}\right)$, $Q\left(at_2, \dfrac{a}{t_2}\right)$ and the origin are collinear.

4. (a) If $y = (x^2 - 1)^n$, prove that :

$$(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0.$$

(b) The radius of curvature ρ at a point on a curve is given by the formula :

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{d^2y/dx^2}$$

Find the radius of curvature at the point 'θ' on the curve defined by $x = 3\cos\theta - \cos^3\theta$, $y = 3\sin\theta - \sin^3\theta$.

5. (a) Integrate :

$$I = \int \frac{x}{(x+1)(x^2 + 4x + 13)}\,dx.$$

(b) Evaluate :

$$I = \int_0^{\pi/2} \frac{1}{2+\cos x}\, dx.$$

6. (a) Solve :

$$\left(1+e^{x/y}\right)dx + e^{x/y}(1-x/y)dy = 0.$$

(b) Solve :

$$x^4 \frac{dy}{dx} + x^3 y + \operatorname{cosec}(xy) = 0.$$

7. (a) The heights and weights of the 10 armymen are given below. In which characteristics are they more variable ?

Height in cm	Weight in kg.
170	75
172	74
168	75
177	76
179	77
171	73
173	76
178	75
173	74
179	75

(b) What do you mean by measures of central tendency ? What should be the characteristics of an ideal measure ? Describe the different situations in which the specific measures of central tendency and of dispersion are the most suitable one.

8. (a) Define probability of an event A. State addition and multiplications laws.

A can hit a target 3 times in 5 shots, B 2 times in 5 shots and C 3 times in 4 shots. They fire a volley. What is the probability that :

(i) two shots hit

(ii) at least two shots hit ?

(b) In a bolt factory, machines A, B and C manufacture 25%, 35% and 40% of the total of their output 5%, 4% and 2% are defective bolts. A bolts is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B or C ?